

# Phase Selection Heuristics for Satisfiability Solvers

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**Abstract.** In general, a SAT Solver based on conflict-driven DPLL consists of variable selection, phase selection, Boolean Constraint Propagation, conflict analysis, clause learning and its database maintenance. Optimizing any part of these components can enhance the performance of a solver. This paper focuses on optimizing phase selection. Although the ACE (Approximation of the Combined lookahead Evaluation) weight is applied to a lookahead SAT solver such as March, so far, no conflict-driven SAT solver applies successfully the ACE weight, since computing the ACE weight is time-consuming. Here we apply the ACE weight to partial phase selection of conflict-driven SAT solvers. This can be seen as an improvement of the heuristic proposed by Jeroslow-Wang (1990). We incorporate the ACE heuristic and the existing phase selection heuristics in the new solver MPhaseSAT, and select a phase heuristic in a way similar to portfolio methods. Experimental results show that adding the ACE heuristic can improve the conflict-driven solvers. Particularly on application instances, MPhaseSAT with the ACE heuristic is significantly better than MPhaseSAT without the ACE heuristic, and even can solve a few SAT instances that remain unsolvable so far.

**Key words:** SAT Solver, conflict-driven DPLL, phase selection heuristics, instance classifying, hard SAT instance.

## 1 Introduction

Satisfiability (SAT) is a classic NP-complete problem, and has applications in numerous fields such as computer aided design, data diagnosis, EDA, logic reasoning, cryptanalysis, planning, equivalence checking, model checking, test pattern generation etc. To address the SAT problem, numerous state-of-the-art solvers have been developed. This problem has been studied for a long time. However, large real-world SAT problems remain unsolvable yet.

Most modern SAT solvers are based on conflict-driven DPLL. The most representative solver in this type of solvers is Precosat [3,4,5], which won a Gold Medal for application category at SAT 2009 competition. Generally speaking, this type of solvers is good at solving application instances. In this paper, we focus on a conflict-driven DPLL-type solver like Precosat. In general, a

conflict-driven DPLL-type solver consists of variable selection, phase selection, BCP (Boolean Constraint Propagation), conflict analysis, clause learning and its database maintenance. Each part has various optimizing strategies. For example, for variable selection, the corresponding optimizing strategy is VSIDS (Variable State Independent Decaying Sum) scheme [16]. To speed up BCP, two watched-literals scheme was proposed. With respect to conflict analysis, a large amount of optimizing work has been done. The research results such as firstUIP (unique implication points), conflict clause minimization, on-the-fly self-subsuming resolution [17], learned clause minimization [18], have been achieved. To maintain effectively clause learning database, recently, Audemard et al. [19] introduced a Glucose style reduce strategy to remove less important learned clauses. Although the phase heuristic is also an important component of modern conflict-driven SAT solvers, the literature on the phase selection is rare. To our best knowledge, up to now, only two phase selection strategies were widely used in conflict-driven SAT solvers. One is the phase heuristic used in RSAT (RSAT heuristic for short) [1]. The other is Jeroslow-Wang heuristic [2]. The basic idea of the RSAT heuristic is to save the previous phase and assign the decision variable to the same value when it is visited once again. The basic idea of Jeroslow-Wang heuristic is to define variable polarity as a phase with the maximum weight. The value of weight of a variable depends on the number of clauses containing that variable and its size. Although Precosat [5] gains the good performance by integrating the RSAT heuristic and Jeroslow-Wang heuristic, it cannot be concluded that there does not exist a better phase heuristic.

The goal of this paper is to find a new phase selection heuristic that improves the existing phase selection heuristics. If we can select always correctly a phase, all satisfiable formulae will be solved in a linear number of decisions. In theory, no perfect phase selection heuristic exists unless  $P=NP$ . In practice, it is possible to develop a phase selection heuristic that significantly reduces the number of decisions in some cases. To achieve this goal, we hope to use a new phase heuristic, which uses some information about the structure of the problem such as the number of variables, the number of XOR clauses etc. The ACE (Approximation of the Combined lookahead Evaluation) weight has been applied successfully to a lookahead SAT solver such as March. However, so far, no conflict-driven SAT solver applies successfully the ACE weight, since computing the ACE weight is time-consuming. Here we apply partially the ACE weight to phase selection of conflict-driven SAT solvers. This can be seen as an improvement of the Jeroslow-Wang heuristic. Based on our empirical observation, the phase selection heuristic based on ACE can enhance the ability of solving some instances. In our solver, the ACE heuristic is only applied under certain circumstances, mainly due to its cost. In the case the ACE heuristic is not suited for, the other heuristics such as Jeroslow-Wang heuristic are applied. To avoid harming the other heuristics when applying the ACE heuristic, we define a set of criteria for selecting the good phase heuristic in a way similar to portfolio methods [7,8,9,10,11] to classify instances. Our method to classify instances uses fewer features, and is simpler than the model-based portfolio method [8]. We build a new SAT solver,

called MPhaseSAT, by integrating multiple phase heuristics including the ACE heuristic and Jeroslow-Wang heuristic.

Empirical results show that adding the ACE heuristic can improve our new solver MPhaseSAT. Particularly for application instances, the improvement is significant. On this category, MPhaseSAT with the ACE heuristic is significantly superior to MPhaseSAT without the ACE heuristic, and even can solve a few SAT instances that remain unsolvable so far. Although the improvement on the crafted category is not so big, MPhaseSAT with the ACE heuristic is still a little better than MPhaseSAT without the ACE heuristic.

## 2 Phase selection heuristics

The decision variable selection is indispensable to conflict-driven SAT solvers. The decision heuristic used in most SAT solvers is a more dynamic and adaptive version of the original zChaff decision heuristic [16]. The phase selection of variable is an inseparable step that follows the decision variable selection, because we must assign each decision variable to a value. The simplest phase selection heuristic is a default heuristic of MiniSAT, in which each decision variable is always assigned to false. To avoid work repetition caused by some independent components, the strategy used in RSAT [1,6] is to assign the decision variable to the same value it has been assigned before. PrecoSAT [5] combines RSAT heuristic [1] and Jeroslow-Wang heuristic [2] to select the phase of the decision variable. Its basic idea is: when the decision variable has not been assigned yet, Jeroslow-Wang heuristic is used. Otherwise, RSAT heuristic is used. The basic idea of the Jeroslow-Wang heuristic is to select the phase of a decision variable by comparing the weights of two phases. Let  $S$  define a set of CNF (Conjunctive Normal Form) clauses. This heuristic defines the weight of  $S$  as

$$W(S) = \sum_{k=1}^{\infty} \frac{n_k}{2^k}$$

where  $n_k$  is the number of clauses of size  $k$  in  $S$ . For a decision variable  $v$ , let  $S_v$  and  $S_{-v}$  be the set of clauses in which  $v$  occurs positively, respectively negatively. If  $W(S_v) > W(S_{-v})$ , this heuristic picks the positive phase, i.e., assign  $v$  to true. Otherwise, it picks the negative phase, i.e., assign  $v$  to false.

Jeroslow and Wang [2] presented a simple analysis on this heuristic, and indicated that when  $W(S) < 1$ ,  $S$  must be satisfiable. In an intuitive sense, as long as we choose always a literal with the maximum weight, which can yield a formula of minimum weight, a formula is most likely to be satisfiable. However, in real applications, this heuristic is not necessarily effective. So far, no state-of-the-art SAT solver uses Jeroslow-Wang heuristic to pick a decision variable. The solver PrecoSAT applies it to only the phase selection of a decision variable, not decision variable selection. In our experiments, we noted that in some cases Jeroslow-Wang heuristic was efficient for the phase selection of a decision variable, but in some cases the new heuristic given below was more efficient.

The new heuristic uses some information about the structure of the problem such as the number of variables, the number of XOR clauses, etc. It defines the weight of a literal as ACE (Approximation of the Combined lookahead Evaluation). The heuristic based on the ACE weight here is called ACE heuristic. The concept of ACE is widely used in lookahead SAT solvers such as March [14], MoRsat [15]. The concept of ACE here is the same as that one used in MoRsat. However, its computation is simpler than that of ACE in March. Let the notation  $\mathcal{F}(x = 0)$  denote the resulting formula after assigning literal  $x$  to false and performing iterative unit propagation.  $\mathcal{F}(x = 1)$  is similar. The ACE weight of a literal  $x$  is defined as

$$\text{ACE}(x, \mathcal{F}, \mathcal{F}') = \sum_{c \in \text{CNF}(x, \mathcal{F})} W_{\text{CNF}}(\text{size}(c, \mathcal{F}')) + \sum_{c \in \text{XOR}(x, \mathcal{F})} W_{\text{XOR}}(\text{size}(c, \mathcal{F}'))$$

where  $\mathcal{F}'$  is either  $\mathcal{F}(x = 0)$  or  $\mathcal{F}(x = 1)$ .  $\text{CNF}(x, \mathcal{F})$  and  $\text{XOR}(x, \mathcal{F})$  are the set of CNF clauses and the set of XOR clauses in formula  $\mathcal{F}$  in which variable  $x$  occurs, respectively, and  $\text{size}(c, \mathcal{F}')$  denotes the length of the clause to which  $c$  is reduced after an iterative unit propagation  $\mathcal{F}'$ , and weight functions  $W_{\text{CNF}}(n)$  and  $W_{\text{XOR}}(n)$  are defined as

$$W_{\text{CNF}}(n) = 5^{2-n}$$

$$W_{\text{XOR}}(n) = 5.5 \times 0.85^n$$

where  $n$  is the length of the reduced clause. The heuristic defined by ACE chooses always a phase with the maximum ACE weight. In some sense, the ACE heuristic can be seen as an improvement of Jeroslow-Wang heuristic defined above.

**Table 1.** Runtime (in seconds) required by MPhaseSAT with different phase selection heuristics to solve SAT problems.

Instance	# var	# clauses	ACE heuristic	PrecoSAT heuristic
cube-11-h13-unsat	455627	1367522	297	8795
schup-l2s-bc56s-1-k391	561371	1778987	302	911
unif2p-p0.7-v3500-c9345-S1832504551	3500	9344	194	454
unif2p-p0.7-v4500-c12015-S1626790907	4500	12014	3679	5771
lksat-n1000-m6860-k4-l4-s1935114289	1000	6860	4502	8360
lksat-n1100-m7545-k4-l4-s310659001	1100	7545	1952	2582

Table 1 shows some examples that benefit from the ACE heuristic. PrecoSAT heuristic means the combination of Jeroslow-Wang heuristic and RSAT heuristic, which is used in the PrecoSAT solver. All six instances used in this experiment are unsatisfiable. The reason why we did not choose any satisfiable instance is to rule out a lucky solving. The first two instances in Table 1 are from application category in SAT 2009 competition. The middle two instances are from random category in SAT 2007 competition. The last two instances are from crafted category in SAT 2009 competition. The SAT solver used is MPhaseSAT, a variant of

CicleSAT [13], which is built on the top of PrecoSAT 465. As shown in Table 1, each category has some instances for which using the ACE heuristic was faster than using PrecoSAT heuristics. There are also many unsuccessful instances. For example, dated-5-15-u, 9dlx\_vliw\_at\_b\_iq2, q\_query\_3\_148\_lambda, etc, on these instances, using ACE the heuristic was slower on the contrary.

In theory, no matter what phase selection heuristic we use, for unsatisfiable instances, the search efficiency should be the same, as long as no restart occurs, the variable decision policy is the same, and learnt clause database is infinitely extended. However, in fact, every state-of-the-art conflict-driven solver has restart policy and the maximum limit of learnt clause database. Therefore, different phase selection policies perform best on different instances. Then, for a specific instance, what is the best phase selection? This is either an adaptive problem or a performance prediction problem.

In order to address better the phase selection problem, we present the following seven phase heuristics, one or multiple ones of which will be used for solving a SAT instance.

1. Jeroslow-Wang heuristic: JW heuristic for short.
2. ACE heuristic: when the search depth is smaller than 30, ACE weight is used. Otherwise, JW weight is used.
3. JW+RSAT heuristic: a combination of Jeroslow-Wang heuristic and RSAT heuristic. Because it is used in PrecoSAT, it is called PrecoSAT heuristic also.
4. PrecoSAT+tail JW heuristic: within the last 20 search depths, only JW heuristics is applied, without RSAT heuristic. In the other search depths, PrecoSAT heuristic is applied.
5. ACE + PrecoSAT heuristic: when the number of decisions is less than 300000, ACE heuristic is applied. Otherwise, PrecoSAT heuristic is applied.
6. PrecoSAT+random heuristic: this is similar to CryptoMiniSat [12] policy. In general, the phases are calculated for each variable according to PrecoSAT heuristic. However, sometime the phase and the decision variable are randomly selected. Our strategy is that the decision variable is randomly selected with the probability of 0.02, and the phase is randomly flipped probability of 1/30.
7. Local search phase heuristic: the state of local search such as the solver TNM [20] is considered as the basis of phase selection. This heuristic chooses always a phase in accordance with the current state of a variable local search algorithm returns.

Unlike JW heuristic, ACE heuristic is dynamic. That is, the ACE score is calculated at each step for selecting the phase of the new decision point. Its computation cost is much more expensive than that of JW heuristic. Therefore, as seen above, the depth where the ACE heuristic is applied is restricted to 30. If the depth is not limited, the ACE heuristic is not suited for large SAT instances, since it is time-consuming.

Another important problem is that we must decide which heuristics are suited for which SAT instances. That is, how do we classify SAT instances? Portfolio

methods are useful to how to classify SAT instances. Xu et al. [9,10] developed the SATzilla solver, using the SAT portfolio method. Their SAT solver is built on the pre-solver regression-based predictors of performance, and was very successful in the 2009 competition. To improve the existing portfolio method, Silverthorn and Miikkulainen [8] considered unobserved variables as latent variables, and presented two latent class models for algorithm portfolio methods: a mixture of multinomial distributions, and a mixture of Dirichlet compound multinomial distributions. Based on the model-based portfolio method, they developed the borg-sat solver [7], which can outperform SATzilla in all categories from their report [8], and was fairly successful in SAT-Race 2010. After investigating these portfolio methods, we found that the portfolio methods seem not to be suited for our phase selection problem. The latest version of SATzilla [11] uses domain-specific knowledge and select manually more than 90 statistics features, such as statistics of the variable-clause graph and of DPLL probes. This is expensive to our task during training. Borg-sat uses minimal domain knowledge. However, when the number of assumed latent classes,  $K$ , is small, the performance of borg-sat deteriorate sharply. In our phase selection task, it is expected that the value of  $K$  is limited to be small, and the performance should still be good. Therefore, we do not select portfolio methods to classify SAT instances.

Instead, we develop a simple method to classify SAT instances. In the simple method, we choose the following instance features.

1. Number of clauses: denoted by  $\#c$ .
2. Number of variables: denoted by  $\#v$ .
3. Ratio of clauses and variables: denoted by  $\#c/\#v$ .
4. Mean search depth to conflict in DPLL probing: denoted by  $E(\#d)$
5. Number of unfixed variables in DPLL probing: denoted by  $U(\#v)$
6. Number of binary clauses: denoted by  $\#bin$ .
7. Number of XOR clauses: denoted by  $\#xor$ .
8. Number of clauses of size 9 or more: denoted by  $L(\#c)$ .

The computation cost of the above features is very cheap. Moreover, most features are static. Unlike SATzilla, we do not make regression analysis. Our model construction is simple. Based on the observation on the behavior of some representative SAT instances, we classify manually SAT instances into some categories by feature information. Each category corresponds to a phase selection heuristic. How to map a category of SAT instances to a phase selection heuristic? This can be done by an adaptive algorithm, which may be described as follows.

- (1) When  $50000 < \#c < 220000$ , in the preprocessing phase, we use PrecoSAT+random heuristic to determine the variable and its phase. This heuristic is suited well for the crypto instances such as the mizh-sha0 family. Our preprocessing is limited to 200000 decisions.
- (2) We set the phase policy to PrecoSAT heuristic in the following cases.
  - a)  $\#xor < 1000$  and  $\#c > 300000$ .
  - b)  $\#xor > 2000$  and  $U(\#v) < 15000$ .

- c)  $\#c/\#v < 6$  and  $\#c/15 > \#v$  and  $\#c/3 < \#bin/2$ .
- d)  $L(\#c) > 5$  and  $L(\#c) < 40$ .
- (3) We apply ACE heuristic in the following cases.
  - a) In the preprocessing,  $\#c < 18000$ .
  - b)  $E(\#d) < 30$ .
  - c)  $\#bin > 400000$ ,  $\#bin > \#c/2$  and  $\#v/20 > U(\#v)$ .
  - d)  $\#xor > 2000$ ,  $\#xor > \#c/12$  and  $U(\#v) < 15000$ .
- (4) We select PrecoSAT +JW tail heuristic in the following cases.
  - a)  $\#xor = 0$ ,  $\#c/\#v > 100$  and  $\#v < 1500$ .
  - b)  $\#xor = 0$ ,  $\#c/\#v > 55$  and  $\#c/\#bin < 0.9$ .
- (5) In the pre-solving, the default heuristic is set to PrecoSAT heuristic. In the other cases, the default heuristic is set to ACE + PrecoSAT heuristic.
- (6) For random category, we use the MoRsat [15] solving technique and local search phase heuristic.

The above rules for classifying instances and constants are mainly based on application instances in SAT competition 2009. Of course, these rules can become more coarse or fine. This depends on a real application. Based on our observation, the above rules are sufficient for application instances in SAT 2009.

Many modern SAT solvers such as PrecoSAT do not usually support XOR gates. Without making any modification, such solvers cannot apply directly our ACE heuristic. However, it is easy to implement the ACE heuristic by detecting XOR gates during preprocessing, adding occurrence lists, and maintaining original clauses and XOR gate database separately.

### 3 Empirical evaluation

Due to the diversity of SAT instances, in many cases, a new heuristic is in conflict with old heuristics. However, the new heuristic proposed in this paper is not so. By handmade refining and classifying, we integrate this new heuristic to a SAT solver. Does this integration harm the entire performance of the solver? This question will be answered by our experiments.

We carried out the experiments with such a platform: Intel Core 2 Quad Q6600 CPU with speed of 2.40GHz and 2GB memory. The instances used in the experiments are from SAT 2009 competition. The timeouts for solving an application instance and a crafted instance were set to 10000 seconds and 5000 seconds, respectively. The SAT solvers used for a comparison are PrecoSAT [5], CryptoMiniSat [12] and MPhaseSAT. PrecoSAT used here is the latest version 465 of PrecoSAT 236, which won a Gold Medal for application category in the SAT competition 2009. CryptoMiniSat is the winner of the Gold Medal in SAT-Race 2010. MPhaseSAT is an improved version of CircleSAT [13], which is a conflict-driven DPLL complete solver based on PrecoSAT. MPhaseSAT consists of preprocessing, pre-solving, conflict-driven DPLL solving and hybrid DPLL solving. The preprocessing is similar to that of the look-ahead solver March [14]. The pre-solving is used to compute the feature of SAT instances in order to

select successfully the best phase heuristic. It tests run with PrecoSAT. Once a heuristics strategy is determined, MPhaseSAT uses that heuristics strategy to solve with an improved PrecoSAT or a hybrid DPLL solver. For the basic principle of hybrid DPLL solving, the reader is referred to MoRsat [15].

**Table 2.** Performance of solvers on 292 application instances in SAT 2009

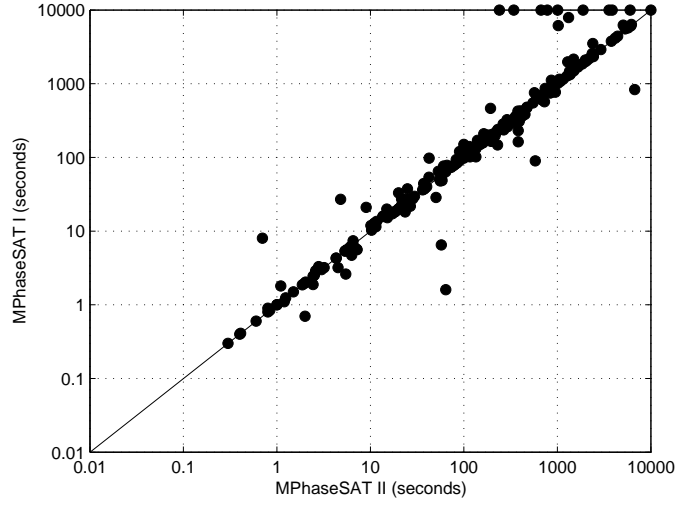
Solver	Instances Solved	Average time (in seconds) per solved instance
PrecoSAT 465	210	734.47
CryptoMiniSat	212	780.40
MPhaseSAT I	218	635.46
MPhaseSAT II	227	645.02

**Table 3.** Performance of solvers on 281 crafted instances in SAT 2009

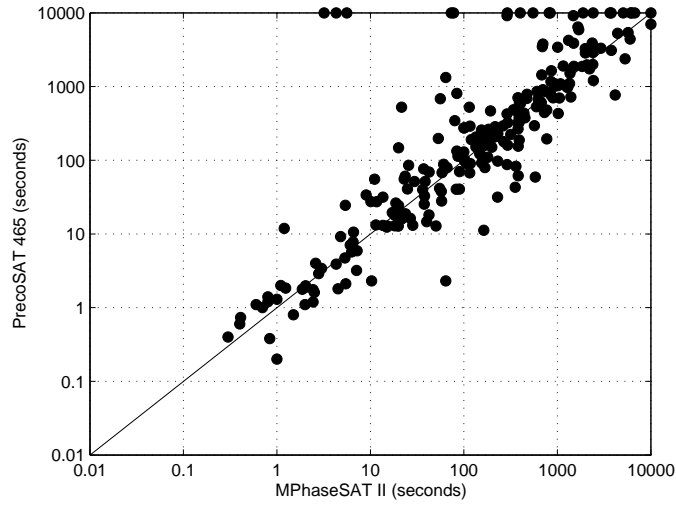
Solver	Instances Solved	Average time (in seconds) per solved instance
PrecoSAT 465	149	391.33
CryptoMiniSat	143	477.96
MPhaseSAT I	176	360.88
MPhaseSAT II	178	296.79

From our empirical results, PrecoSAT 465 outperformed indeed the previous version PrecoSAT 236. In terms of the number of instances solved, PrecoSAT 465 and 236 solved 210 and 204 out of 292 instances in the application category, respectively. As shown In Table 2, CryptoMiniSat solved 212 instances. Except for these solved instances, due to out of memory, 5 instances were not solved by CryptoMiniSat. If memory is sufficient, CryptoMiniSat could solve 217 instances. To measure efficiency of the new phase heuristic, we divided MPhaseSAT into two versions. One is the version with multiple phase heuristics, which integrates the new phase heuristic. The other is the version with single phase heuristic, which uses only PrecoSAT heuristic. MPhaseSAT with single phase heuristic is MPhaseSAT I for short. MPhaseSAT with multiple phase heuristics is MPhaseSAT II for short. As shown in Table 2, for the application category, MPhaseSAT I and II solved 218 and 227 out of 292 instances, respectively. This result reveals that adding the new phase heuristic can improve significantly the performance of the solver. The reason why MPhaseSAT I without ACE heuristic was also better than CryptoMiniSat is because it benefits from the combination of pre-solving technique and hybrid DPLL technique. Because the techniques are not the focus of this paper, we here omitted their implementation details. The virtual best solver in the SAT 2009 competition, which is defined as a theoretical





**Fig. 1.** Comparing the runtimes of MPhaseSAT I and MPhaseSAT II on application instances from SAT 2009.



**Fig. 2.** Comparing the runtimes of MPhaseSAT II and PrecoSAT 465 on application instances from SAT 2009.

solver which returns the best answer provided by one of all the submitted solvers, solved 229 application instances, 225 out of which were solved by MPhaseSAT II. Two instances gss-24-s100 and eq.atree.braun.12 are not included by the 229 instances, which cannot be solved in a reasonable time by any solver so far. Gss-24-s100 was solved by both MPhaseSAT I and II. But eq.atree.braun.12 was solved by only MPhaseSAT II. This is due to the application of the ACE phase heuristic. From this empirical results, it is easy to see that MPhaseSAT II approaches very much the performance of the virtual best solver.

We tested also the effectiveness of the ACE heuristic on the crafted instances from SAT 2009. Because the rules for classifying instances are determined mainly according to the behaviour of our solver on the application category, the ACE heuristic was not very effective on the crafted category in this experiment. As shown in Table 3, MPhaseSAT II is a little better than MPhaseSAT I, since they solved 178 and 176 out of 281 instances, respectively. However, they are much better than the other two solvers. CryptoMiniSat and PrecoSAT solved 143 and 149 out of 281 instances. Notice, the solver clasp, the champion of this category, solved 156 instances. Therefore, both CryptoMiniSat and PrecoSAT are not good at the crafted category. Nevertheless, the excellent behaviour of MPhaseSAT on the crafted category is mainly due to the other technique development including the new at-most-one encoding technique [21], not the phase heuristic given here.

Figures 1 and 2 show a log-log scatter plot comparing the runtimes of MPhaseSAT I and MPhaseSAT II, and the runtimes of MPhaseSAT II and PrecoSAT 465, respectively. The instances in Figures 1 and 2 are from the application category at SAT 2009. The climax (10000,10000) means that the instances on that point were not solved by any of two solvers. Whether in Figure 1 or in Figure 2, except for the points that were not solved by MPhaseSAT I nor PrecoSAT 465, most of points are centralised at the nearby diagonal. This demonstrates that adding the new phase heuristic has no strong impact on solving the instances that is not suited for the new phase heuristic. In fact, this is verified by the average runtime per solved instance shown in Table 2, since the difference between the average runtimes of MPhaseSAT I and MPhaseSAT II is very small.

If the performance of a single solver is equivalent to that of the virtual best solver, it means than all the optimizing techniques can be integrated in a SAT solver. Nevertheless, now it seems impossible to do this. The reason why MPhaseSAT on the application category approached the virtual best solver is because the training set for establishing the rules were also application instances. Crafted instances were not used for the training set. On the crafted category, the performance of MPhaseSAT did not approach that of the virtual best solver. This is reflected by the fact the virtual best solver solved 187 instances in the crafted category at SAT 2009, 166 out of which were solved by MPhaseSAT II. The other 12 instances solved by MPhaseSAT II remains unsolvable so far.

Like the other conflict-driven solvers, MPhaseSAT is not good at the random category. So here we do not discuss the performance of MPhaseSAT on the random category.

## 4 Conclusions

By integrating the ACE phase selection heuristic and the existing heuristics such as Jeroslow-Wang heuristic and RSAT heuristic, we have built a new SAT solver called MPhaseSAT. Surprisingly, MPhaseSAT can not only outperform PrecoSAT, the champion of the application category in SAT 2009, and Crypto-MiniSat, the champion of SAT-Race 2010, but also approaches the virtual best solver in terms of the solving ability. Moreover, it solved a few SAT instances that remain unsolvable so far.

MPhaseSAT is built on the top of the known better solver. Hence, its performance depends heavily on that of the known solvers. Because of the limit of the known technique, perhaps, the usefulness of the new phase selection heuristic has not yet been tapped sufficiently. To obtain a great breakthrough, we must devise a new solving technique different from the existing one.

In this paper, a SAT problem is envisioned as a portfolio. During the entire solving process, a SAT problem uses only one phase heuristic. If a problem consists of multiple independent sub-problems, and each sub-problem has a different portfolio, the classifier of SAT instances given in this paper will fail, since it cannot switch between different sub-problems. Therefore, the adaptive switching technique among different phase heuristics needs to be studied further.

The ACE heuristic is time-consuming, and much more expensive than the existing phase selection heuristics. It is a subject that's worth studying how to simplify the ACE heuristic and reduce its computation cost.

Whether in theory or in practice, we believe that the phase heuristics known so far are not certainly the best, and there exists certainly a better phase heuristics. It is a very valuable research topic how to find out a better phase heuristics.

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